

Abstract for format: talk

Topic: application

Online Learning Module Complex Numbers with Interactive Exercises and Visualizations

Karin Landenfeld, Alexander Steudel, David-Hauke Wedler, Hanna Werner, Emira Zorgati
Faculty of Engineering and Computer Science, Hamburg University of Applied Sciences,
Germany,

Email: karin.landenfeld@haw-hamburg.de

In our contribution we would like to present an online learning module about complex numbers developed at the Hamburg University of Applied Sciences, in which we use JSXGraph in various learning elements. These learning elements are used to illustrate the results, demonstrate the calculations or interactively apply what has been learned.

Complex numbers are fundamental components of an electrical engineering degree. The understanding of complex numbers, their calculations and relations often causes difficulties for students. This is partly due to the fact that the number range of complex numbers was not taught in school before and partly to the abstract nature of the imaginary unit 'j'. Furthermore, the applicability of complex numbers for oscillation calculations and complex AC calculations is difficult for students to understand. The developed learning module is intended to support students in learning and understanding. It is implemented in the Moodle online learning environment www.viamint.de of the Hamburg University of Applied Sciences and can be used freely by students and teachers both in mathematics classes and in lectures on the fundamentals of electrical engineering.

(1) Structure of the learning module

The learning module consists of various learning sequences (see Figure 1) in which short explanatory videos and small interactive tasks (see (2)) are arranged alternately. Within the videos, the learning content and calculations are visualized with an applet especially developed for this module (see (3)) for better understanding. Students can also use the applet for their own practice and learning. In-depth exercises are available at the end of a learning sequence to help the students to consolidate and test their knowledge (see (4)). An introductory motivational video and a detailed collection of formulas complete the learning module.

Learning Modul Complex Numbers

contents

Komplexe Zahlen (Beta-Version)

1. Grundlagen und Darstellungsformen
 - 1.1 Grundlagen
 - 1.2 Darstellungsformen
2. Rechnen mit komplexen Zahlen
 - 2.1 Grundrechenarten
 - 2.2 Weitere Rechenarten
3. Komplexe Zahlen in der Elektrotechnik
4. Abschlusstest

interactive applet

Ergänzende Materialien
Applet: Darstellung von komplexen Zahlen

explanation video

small interactive exercise

Aufgabe 4: konjugiert komplexe Zahlen

Bestimme für die komplexen Zahlen z_1 und z_2 jeweils die konjugiert komplexe Zahl. Ziehe dafür die Punkte z_1^* und z_2^* im untenstehenden Applet so, dass sie das konjugiert komplexe Gegenstück zu den gegebenen komplexen Zahlen bilden.

$z_1 = 3 + 2j$
 $z_2 = -1 - j$

learning sequence 1.1

in-depth exercises

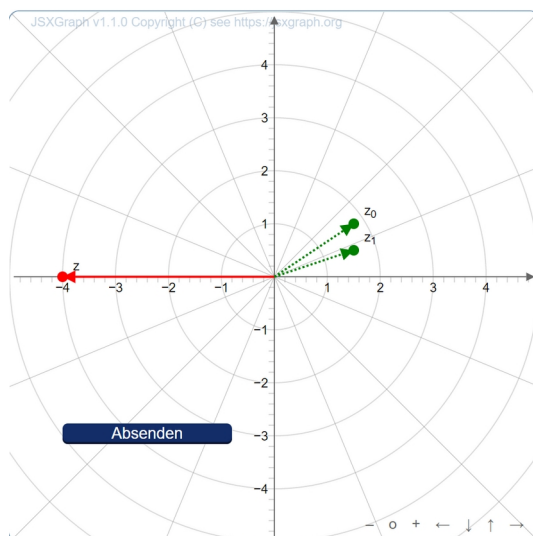
Figure 1: Structure of the learning module complex numbers with an exemplary learning sequence and some examples of learning elements

(2) Small interactive exercises between the learning videos

The exercises after an explanatory video give the students the opportunity to become active themselves and to directly check what they have learned. The tasks are implemented in HTML and JavaScript, allowing an easy integration of JSXGraph interactions. The results of the interaction are evaluated via HTML and if necessary, with a connected computer algebra library.

Wurzel einer komplexen Zahl

Berechnet die zweite Wurzel der eingezeichneten komplexen Zahl z .
Skizziert die verschiedenen Lösungen z_0 und z_1 in der Gauß'schen Zahlenebene beginnend bei z_0 mit dem kleinsten positiven Winkel:



Wurzel einer komplexen Zahl

Berechnet die zweite Wurzel der eingezeichneten komplexen Zahl z .
Skizziert die verschiedenen Lösungen z_0 und z_1 in der Gauß'schen Zahlenebene beginnend bei z_0 mit dem kleinsten positiven Winkel:

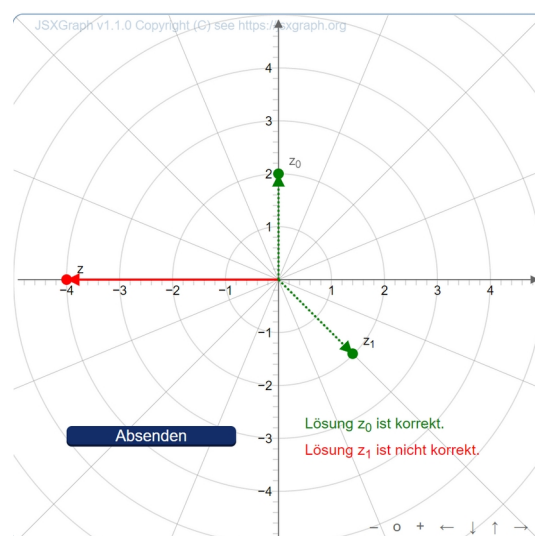


Figure 2: Example of a short JSXGraph exercise and feedback after checking the inputs

(3) Applet complex numbers

The applet was developed using JSXGraph and HTML/JavaScript elements to suit the needs of this learning module. Complex numbers can be entered both via numeric input fields and in the graphic visualization by positioning the complex pointer. The various arithmetic operations with complex numbers as well as the option of the representation in Cartesian or polar coordinates can be selected. The result of the calculation is displayed both as a numerical value and in the graphic visualization.

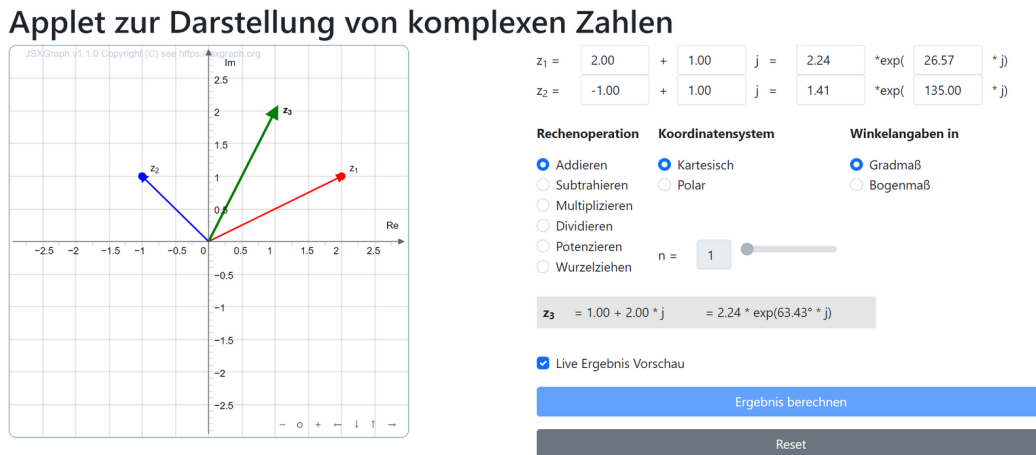
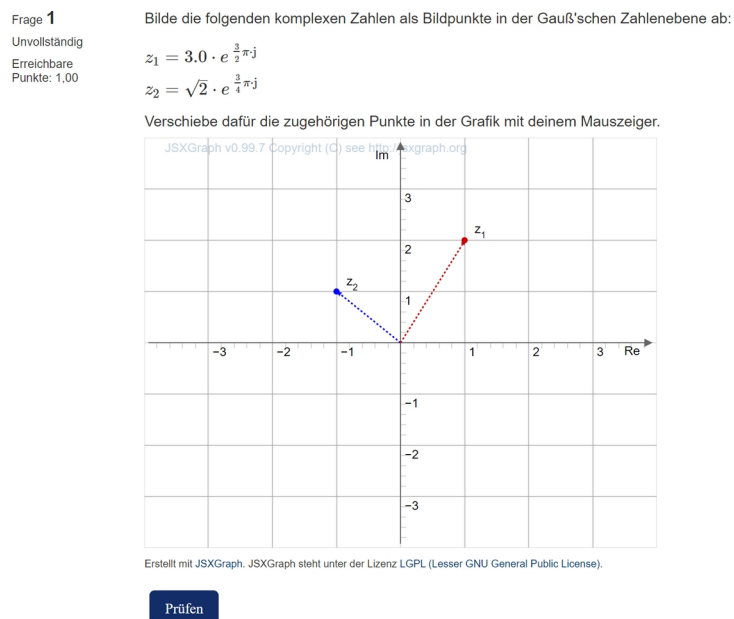


Figure 3: User interface of the applet

(4) In-depth exercises at the end of a learning sequence

These exercises are intended to deepen the understanding of the learned content. They are implemented with the Moodle exercise type STACK, with which the integration of JSXGraph elements and a computer algebra system is possible. JSXGraph is used here both for interactive inputs and for visualizing the sample solution of the randomized tasks. The graphical input of results in a JSXGraph visualization can be submitted to the computer algebra system for verification using STACK, which then enables evaluation and specific feedback. An example is given in the following figure 4.



Deine Antwort ist teilweise korrekt.

z_1 ist leider falsch eingetragen.

Tipp: Rechne die komplexe Zahl z_1 in die algebraische Form um. Am Real- und Imaginärteil kannst du dann die kartesischen Koordinaten des Punktes in der Gauß'schen Zahlenebene ablesen.

z_2 ist richtig eingetragen.

Musterlösung:

Die Beträge sowie die Winkel zwischen der x -Achse und dem Zeiger zu z_1 bzw. z_2 kannst du direkt aus der Exponentialdarstellung der beiden komplexen Zahlen ablesen:

Für $z_1 = 3.0 \cdot e^{\frac{3}{2}\pi j}$ ist

- der Betrag $|z_1| = r_1 = 3.0$,
- der Winkel $\varphi_1 = \frac{3\pi}{2} = 270^\circ$.

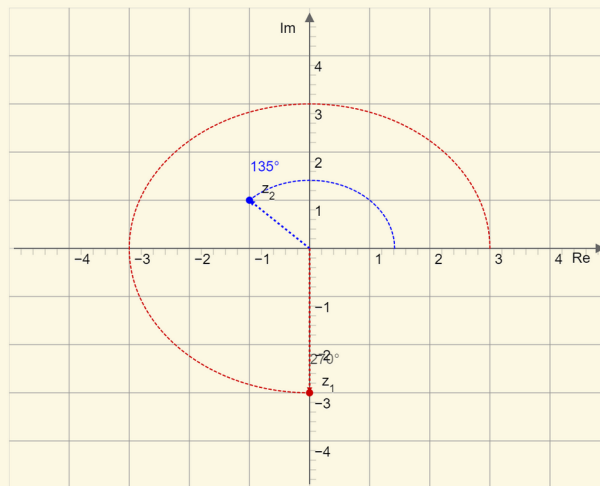
Für $z_2 = \sqrt{2} \cdot e^{\frac{3}{4}\pi j}$ ist

- der Betrag von $|z_2| = r_2 = \sqrt{2}$,
- der Winkel $\varphi_2 = \frac{3\pi}{4} = 135^\circ$.

Um die Zahlen in der Gauß'schen Zahlenebene einzutragen, ist es aber am einfachsten, wenn du die beiden komplexen Zahlen zunächst in die algebraische Form umwandelst. Dabei wird der Realteil mittels $x = r \cdot \cos(\varphi)$ und der Imaginärteil mittels $y = r \cdot \sin(\varphi)$ berechnet. Hieraus ergeben sich die Koordinaten von z_1 und z_2 in der Gauß'schen Zahlenebene:

$$\begin{aligned} \text{Für } z_1: \quad x_1 &= 3.0 \cdot \cos\left(\frac{3\pi}{2}\right) = 0 \\ y_1 &= 3.0 \cdot \sin\left(\frac{3\pi}{2}\right) = -3.0 \end{aligned}$$

$$\begin{aligned} \text{Für } z_2: \quad x_2 &= \sqrt{2} \cdot \cos\left(\frac{3\pi}{4}\right) = -1 \\ y_2 &= \sqrt{2} \cdot \sin\left(\frac{3\pi}{4}\right) = 1 \end{aligned}$$



Erstellt mit JSXGraph. JSXGraph steht unter der Lizenz LGPL (Lesser GNU General Public License).

Figure 4: Example for Moodle question with JSXGraph

The development of the learning module was funded by the Hamburg Open Online University (HOOU). The learning module will be published at www.hoou.de under the Creative Commons license CC-BY-SA. Free use is possible via the online learning environment www.viamint.de at the Hamburg University of Applied Sciences. A beta version is available for use at the moment.